One-loop Self-energies in the Electroweak Model with Nonlinearly Realized Gauge Group [1]

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Abstract

We evaluate at one loop the selfenergies in generic D dimensions for the W, Z mesons in the Electroweak model where the gauge group is nonlinearly realized. In this model the Higgs boson parameters are absent, while a second mass parameter appears together with a scale for the radiative corrections. We estimate these parameters in a simplified fit on leptons and gauge bosons data. We check physical unitarity and the absence of infrared divergences. Landau gauge is used.

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1 Introduction

A consistent strategy for the all orders subtraction of the divergences in nonlinearly realized gauge theories has been recently developed [1]-[5] by extending some tools originally devised for the nonlinear sigma model in the flat connection formalism [6]. The approach relies on the local functional equation for the 1-PI vertex functional [6] (encoding the invariance of the group Haar measure under local left transformations), the weak power-counting theorem [7] and the pure pole subtraction of properly normalized 1-PI amplitudes [8]. This scheme of subtraction fulfills all the relevant symmetries of the vertex functional. Physical unitarity is established as a consequence of the validity of the Slavnov-Taylor identity [9].

This strategy has been first applied to the nonlinearly realized $SU(2)$ massive Yang-Mills theory [3]. The full set of one-loop counterterms and the self-energy have been obtained in [4].

The extension to the electroweak model based on the nonlinearly realized $SU_L(2) \otimes U_Y(1)$ gauge group introduces a number of additional non-trivial features [1, 2]. The direction of the Spontaneous Symmetry Breaking fixes the linear combination of the hypercharge and of the third generator of the weak isospin giving rise to the electric charge. Despite the fact that both the hypercharge and the $SU_L(2)$ symmetry are non linearly realized, the Ward identity for the electric charge has a linear form on the vertex functional.

The anomalous couplings are forbidden by the $U(1)_Y$ invariance together with the weak power-counting. However, two independent mass parameters for the vector mesons are allowed. Thus the ratio of the vector meson masses is not anymore given by the Weinberg angle.

As a first step toward a detailed analysis of the radiative corrections of the nonlinearly realized electroweak theory, we provide in this paper the vector meson self-energies in the one-loop approximation.

The dependence of the self-energies on the second mass parameter is important in order to establish a comparison with the linear realization of the electroweak group based on the Higgs mechanism.

We provide a rough estimate both of the extra mass parameter and of the scale of the radiative corrections. We fix some of the parameters on measures taken at (almost) zero momentum transfer, while the one-loop corrections are confronted with measures at the resonant value of the vector bosons energies. The resulting values are challenging: the departure from

the Weinberg relation between the vector meson mass is very small and the scale of the radiative corrections is of the order of hundred GeV.

This is the aim of the work: to provide the amplitudes in D dimensions for future high order computations and to provide a preliminary assessment of the predictivity of the Electroweak Model based on the nonlinearly realized gauge group including the one-loop self-energies corrections. Electroweak physics is described with very reasonable parameters (the second mass parameter and the scale of the radiative corrections).

The cancellations among unphysical states required by Physical Unitarity can be easily traced out. The physical amplitudes are shown to be free of infrared divergences. It is remarkable also that they do not depend from the Spontaneous Breakdown of Symmetry parameter v. This fact has been discussed in Refs. [1], [2] and [4].

The computation is done in the symmetric formalism on the $SU(2)_L$ flavor basis. This choice simplifies greatly both the Feynman rules and the actual computation; in fact symmetry arguments turn out to be very useful in the calculation of the invariant functions. The symmetric formalism puts emphasis on the fact that the entering parameters are not renormalized (e.g. as in the on-shell renormalization procedure) and are fixed at the end, by means of the comparison with the experimental data. Moreover the symmetric formalism makes the underlying symmetric structure encoded by the local functional equation more transparent.

2 Feynman rules

The classical action is written in order to establish the Feynman rules. We omit all the external sources which are needed in order to subtract the divergences at higher loops. We refer to the previous publications [1], [2] where the procedure is described at length. The field content of the electroweak model based on the nonlinearly realized $SU(2)_L \otimes U(1)$ gauge group includes the $SU(2)_L$ connection $A_\mu = A_{a\mu}\frac{\tau_a}{2}$ ($\tau_a,\ a = 1, 2, 3$ are the Pauli matrices), the $U(1)$ connection B_μ, the fermionic left doublets collectively denoted by L and the right singlets, i.e.

$$L \in \left\{ \begin{pmatrix} l^u_{Lj} \\ l^d_{Lj} \end{pmatrix}, \begin{pmatrix} q^u_{Lj} \\ V_{jk} q^d_{Lk} \end{pmatrix}, \quad j, k = 1, 2, 3 \right\},$$

$$R \in \left\{ \begin{pmatrix} l^u_{Rj} \\ l^d_{Rj} \end{pmatrix}, \begin{pmatrix} q^u_{Rj} \\ q^d_{Rj} \end{pmatrix}, \quad j = 1, 2, 3 \right\}. \tag{1}$$

In the above equation the quark fields $(q^u_j, j = 1, 2, 3) = (u, c, t)$ and $(q^d_j, j = 1, 2, 3) = (d, s, b)$ are taken to be the mass eigenstates in the tree-level lagrangian; V_{jk} is the CKM matrix. Similarly we use for the leptons the notation $(l^u_j, j = 1, 2, 3) = (\nu_e, \nu_\mu, \nu_\tau)$ and $(l^d_j, j = 1, 2, 3) = (e, \mu, \tau)$. The single left doublets are denoted by L^l_j, $j = 1, 2, 3$ for the leptons, L^q_j, $j = 1, 2, 3$ for the quarks. Color indexes are not displayed.

One also introduces the $SU(2)$ matrix Ω

$$\Omega = \frac{1}{v}(\phi_0 + i\phi_a\tau_a), \quad \Omega^\dagger\Omega = 1 \Rightarrow \phi_0^2 + \phi_a^2 = v^2 . \tag{2}$$

The mass scale v gives ϕ the canonical dimension at $D = 4$. We fix the direction of Spontaneous Symmetry Breaking by imposing the tree-level constraint

$$\phi_0 = \sqrt{v^2 - \phi_a^2} . \tag{3}$$

The $SU(2)$ flat connection is defined by

$$F_\mu = i\Omega\partial_\mu\Omega^\dagger . \tag{4}$$

2.1 Classical Action

Discarding the neutrino mass terms, the classical action for the nonlinearly realized $SU(2) \otimes U(1)$ gauge group with two independent mass parameters for the vector mesons can be written as follows, where the dependence on Ω is explicitly shown:

$$\begin{aligned}
S = \Lambda^{(D-4)} \int d^D x \Bigg(& 2\,Tr\left\{ -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right\} \\
& + M^2\, Tr\left\{ (gA_\mu - \frac{g'}{2}\Omega\tau_3 B_\mu\Omega^\dagger - F_\mu)^2 \right\} \\
& + M^2 \frac{\kappa}{2}\Big(Tr\{(gA_\mu - \frac{g'}{2}\Omega\tau_3 B_\mu\Omega^\dagger - F_\mu)\tau_3\} \Big)^2 \\
& + \sum_L \left[\bar{L}(i\,\partial\!\!\!/ + g\,A\!\!\!/ + \frac{g'}{2}Y_L\,B\!\!\!/)L + \sum_R \bar{R}(i\,\partial\!\!\!/ + \frac{g'}{2}(Y_L + \tau_3)\,B\!\!\!/)R \right] \\
& + \sum_j \Big[m_{l_j}\ \bar{R}^l_j \frac{1-\tau_3}{2}\Omega^\dagger L^l_j - m_{q^u_j}\ \bar{R}^q_j \frac{1+\tau_3}{2}\Omega^\dagger L^q_j \\
& + m_{q^d_k} V^\dagger_{kj}\ \bar{R}^q_k \frac{1-\tau_3}{2}\Omega^\dagger L^q_j + h.c. \Big] \Bigg) .
\end{aligned} \tag{5}$$

In D dimensions the doublets L and R obey

$$\gamma_D L = -L \quad \gamma_D R = R, \tag{6}$$

being γ_D a gamma matrix that anticommutes with every other γ^μ.

The non-Abelian field strength $G_{\mu\nu}$ is defined by

$$G_{\mu\nu} = G_{a\mu\nu}\frac{\tau_a}{2} = (\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g\epsilon_{abc}A_{b\mu}A_{c\nu})\frac{\tau_a}{2}\,, \tag{7}$$

while the Abelian field strength $F_{\mu\nu}$ is

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,. \tag{8}$$

In the above equation the phenomenologically successful structure of the couplings has been imposed by hand. However the same structure is required by the Weak Power Counting requirement as discussed in Ref. [2].

2.2 Gauge-fixing

In order to set up the framework for the perturbative quantization of the model, the classical action in eq.(5) needs to be gauge-fixed. The ghosts associated with the $SU(2)_L$ symmetry are denoted by c_a. Their antighosts are denoted by $\bar{c}_a$, the Nakanishi-Lautrup fields by b_a. It is also useful to adopt the matrix notation

$$c = c_a\frac{\tau_a}{2}\,, \qquad b = b_a\frac{\tau_a}{2}\,, \qquad \bar{c} = \bar{c}_a\frac{\tau_a}{2}\,. \tag{9}$$

The abelian ghost is c_0, the abelian antighost $\bar{c}_0$ and the abelian Nakanishi-Lautrup field b_0.

For the sake of simplicity we deal here with the Landau gauge. All external sources are dropped out since they are not relevant for the present work. The complete set of external sources is provided in Ref. [2]. Then the gauge-fixing part of the classical action is

$$\begin{aligned} & S_{\mathrm{GF}} \\ & = \Lambda^{(D-4)}\int d^Dx\left(b_0\partial_\mu B^\mu - \bar{c}_0\Box c_0 + 2Tr\,\left\{b\partial_\mu A^\mu - \bar{c}\partial^\mu D[A]_\mu c\right\}\right) \end{aligned} \tag{10}$$

2.3 Bosons symmetric formalism

The bilinear part of the boson sector is

$$
\begin{aligned}
&\frac{M^2}{2}\left[(gA_{a\mu} - g' B_\mu \delta_{3a} - \frac{2}{v}\partial_\mu\phi_a)^2 + \kappa(G\,Z_\mu - \frac{2}{v}\partial_\mu\phi_3)^2\right] \\
&+b_0\partial_\mu B^\mu + b_a\partial_\mu A_a^\mu \\
&= M^2 g^2 |W_\mu^+ - \frac{2}{vg}\partial_\mu\phi^+|^2 + b^+\partial^\mu W_\mu^- + b^-\partial^\mu W_\mu^+ \\
&+\frac{M^2}{2}(G^2)(1+\kappa)\left[Z - \frac{2}{vG}\partial_\mu\phi_Z\right]^2 + b_Z\partial^\mu Z_\mu + b_A\partial^\mu A_\mu
\end{aligned}
\tag{11}
$$

We use the notations

$$
\begin{aligned}
&G = \sqrt{g^2 + g'^2}, \qquad c = \frac{g}{G}, \qquad s = \frac{g'}{G} \\
&M_W = gM, \qquad M_Z = \sqrt{(1+\kappa)}GM
\end{aligned}
\tag{12}
$$

and

$$
W^+ = \frac{1}{\sqrt{2}}(A_1 - iA_2),\ Z = \frac{1}{G}(gA_3 - g'B),\ A = \frac{1}{G}(g'A_3 + gB) \tag{13}
$$

In the Landau gauge the propagator matrix for the bilinear form

$$
-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(V_\mu - \frac{2}{v}\partial_\mu\phi_3)^2 + b\partial_\mu V^\mu \tag{14}
$$

is given by

$$
\left(\begin{array}{cccc}
 & V_\nu & b & \phi \\
V_\mu & \frac{-i}{p^2-m^2}(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) & -\frac{p_\mu}{p^2} & 0 \\
b & \frac{p_\nu}{p^2} & 0 & -i\frac{v}{2p^2} \\
\phi & 0 & -i\frac{v}{2p^2} & \frac{v^2}{4m^2}\frac{i}{p^2}
\end{array}\right). \tag{15}
$$

In the symmetric notation we have

$$
\begin{aligned}
\langle (A_1A_1)_+\rangle &= \langle (A_2A_2)_+\rangle = \langle (W^+W^-)_+\rangle \\
\langle (A_1A_2)_+\rangle &= \langle (A_1A_3)_+\rangle = \langle (A_2A_3)_+\rangle = 0 \\
\langle (A_3A_3)_+\rangle &= \frac{1}{G^2}\langle\Big((gZ + g'A)(gZ + g'A)\Big)_+\rangle \\
&= \frac{1}{G^2}\Big(g^2\Delta_{M_Z} + g^{'2}D\Big) \\
\langle (A_3B)_+\rangle &= \frac{1}{G^2}\langle\Big((gZ + g'A)(-g'Z + gA)\Big)_+\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{gg'}{G^2}\Big(-\Delta_{M_Z}+D\Big) \\
\langle (BB)_+\rangle &= \frac{1}{G^2}\langle\Big((-g'Z+gA)(-g'Z+gA)\Big)_+\rangle \\
&= \frac{1}{G^2}\Big(g'^2\Delta_{M_Z}+g^2D\Big),
\end{aligned}
\tag{16}
$$

where we used a short hand notation, e.g.

$$
\begin{aligned}
&\Delta_{M_Z}\longrightarrow \frac{-i}{p^2-M_Z^2}(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}) \\
&D\longrightarrow \frac{-i}{p^2}(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}).
\end{aligned}
\tag{17}
$$

2.4 Bosons trilinear couplings

For the one-loop calculation of the vector boson self-energies one needs the usual Feynman rules and the trilinear couplings generated by the two mass-invariants in eq. (5). The first mass invariant generates the trilinear couplings

$$
\begin{aligned}
&\frac{M^2}{2}Tr\left\{2\Big(gA_\mu-\frac{g'}{2}\Omega B_\mu T_0\Omega^\dagger-F_\mu\Big)^2\right\}\Big|_{\text{TRILINEAR}} \\
&=\frac{M^2}{2}\Big\{-4\frac{G}{v^2}Z_\mu\epsilon_{3bc}\partial^\mu\phi_b\phi_c-4\frac{g}{v^2}\sum_{a=1,2}A_{a\mu}\epsilon_{abc}\partial^\mu\phi_b\phi_c \\
&+4\frac{gg'}{Gv}(-g'Z+gA)_\mu A_a^\mu\epsilon_{3ab}\phi_b-8\frac{g'}{Gv^2}(-g'Z+gA)_\mu\epsilon_{3bc}\partial^\mu\phi_b\phi_c\Big\}
\end{aligned}
\tag{18}
$$

The second mass invariant yields

$$
\begin{aligned}
&\frac{\kappa M^2}{2}\Big(Tr\Big\{g\Omega^\dagger A_\mu\Omega T_0-\frac{g'}{2}B_\mu+i\Omega^\dagger\partial_\mu\Omega T_0\Big\}\Big)^2\Big|_{\text{TRILINEAR}} \\
&=\frac{\kappa M^2}{2}\Big(4\frac{1}{v^2}G\,Z_\mu\epsilon_{3bc}(\partial^\mu\phi_b)\phi_c-4g\frac{1}{v}G\,Z^\mu A_{a\mu}\epsilon_{ab3}\phi_b \\
&+\frac{8}{v^2}gA_{a\mu}\partial^\mu\phi_3\epsilon_{3ab}\phi_b\Big).
\end{aligned}
\tag{19}
$$

We put everything together

$$
\begin{aligned}
&S\Big|_{\text{BOSON TRILINEAR}} \\
&=\frac{M^2}{2}\Big\{-4\frac{g}{v^2}\sum_{a=1,2}A_{a\mu}\epsilon_{abc}\partial^\mu\phi_b\phi_c+8\kappa\frac{g}{v^2}A_{a\mu}\partial^\mu\phi_3\epsilon_{3ab}\phi_b
\end{aligned}
$$

$$
+4\frac{G}{v^2}\Big(\frac{-g^2+g'^2}{G^2}+\kappa\Big)Z_\mu\epsilon_{3bc}\partial^\mu\phi_b\phi_c-4\frac{gG}{v}(\frac{g'^2}{G^2}+\kappa)Z_\mu A_a^\mu\epsilon_{3ab}\phi_b
$$

$$
-8\frac{gg'}{Gv^2}A_\mu\epsilon_{3bc}\partial^\mu\phi_b\phi_c+4\frac{g^2g'}{Gv}A_\mu A_a^\mu\epsilon_{3ab}\phi_b\Bigg\} \tag{20}
$$

2.5 Fermions contribution

The evaluation of the fermion loops requires a rule on how to handle the γ_5 in dimensional regularization. Our mechanism of removal of divergences is based on a regularization that respects the local gauge invariance therefore γ_5 must anticommute with any γ_μ. At one loop this is possible, as it is well known, since there are no chiral anomalies. For higher loop calculation any trace involving γ_5 must be considered as an independent amplitude up to the end of the subtraction procedure.

The fermion contribution can be easily casted into a global formula

$$
\Gamma_{\mu\nu}[ABST]\equiv-\langle 0|\Bigg(\bar\psi(x)\gamma_\mu(A+B\gamma_5)\psi(x)\bar\psi(0)\gamma_\nu(S+T\gamma_5)\psi(0)\Bigg)_+|0\rangle
$$

$$
=-Tr\Bigg\{\int\frac{dp}{(2\pi)^D}\int\frac{dq}{(2\pi)^D}\gamma_\mu(A+B\gamma_5)\frac{\not p+\not q+m}{(q+p)^2-m^2}e^{ipx}
$$

$$
\gamma_\nu(S+T\gamma_5)\frac{\not q+M}{q^2-M^2}\Bigg\} \tag{21}
$$

where A,B,S,T are matrix elements corresponding to the flavor and the color of the fermions with mass m and M and can be obtained from the classical action (5). In particular the neutral sector is

$$
GZ^\mu\bar\psi\Bigg[\Bigg(\frac{\tau_3}{4}-s^2Q\Bigg)\gamma_\mu-\frac{\tau_3}{4}\gamma_\mu\gamma_5\Bigg]\psi+eA^\mu\bar\psi Q\gamma_\mu\psi,\qquad e\equiv\frac{gg'}{G}. \tag{22}
$$

One then gets the transverse part of the contribution of the fermions (for the notations see Appendix A)

$$
\Gamma_T[ABST]=4\frac{1}{D-1}Tr\Bigg\{i(AS+BT)\frac{(2-D)}{2}\Bigg(\Delta_m+\Delta_M\Bigg)
$$

$$
+H(m,M)\Bigg[mM(AS-BT)D+\frac{(2-D)}{2}(AS+BT)\Bigg(-p^2+M^2+m^2\Bigg)\Bigg]
$$

$$
-\frac{1}{p^2}\frac{1}{2}(AS+BT)(m^2-M^2)i(\Delta_M-\Delta_m)
$$

$$
-\frac{1}{p^2}H(m,M)\Bigg[mM(AS-BT)p^2
$$

$$+\frac{1}{2}(AS+BT)\Big((m^2-M^2)^2-p^2(m^2+M^2)\Big)\Big]\Big\} \tag{23}$$

and the longitudinal part of the contribution of the fermions

$$\Gamma_L[ABST]=\frac{4}{p^2}Tr\Big\{\frac{1}{2}(AS+BT)\Big(i(-m^2+M^2)\Delta_m+i(m^2-M^2)\Delta_M\Big)$$

$$+H(m,M)\Big[mM(AS-BT)p^2$$

$$+\frac{1}{2}(AS+BT)\Big(M^4+m^4-2m^2M^2-p^2(m^2+M^2)\Big)\Big]\Big\}. \tag{24}$$

3 Self-energy amplitudes in D dimensions

The presentation of the results (Landau gauge in D dimension) is as follows. First we report the result of the calculation for the transverse and for the longitudinal parts

$$\Sigma_{\mu\nu}=(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2})\Sigma_T+\frac{p_\mu p_\nu}{p^2}\Sigma_L. \tag{25}$$

For each of them we report the contributions of the single graphs. Subsequently we evaluate the diagonal amplitudes Σ_T on-shell, where we discuss the validity of physical unitarity and the absence of any infrared singularity. Finally the on-shell amplitudes are taken at $D=4$. We omit, for sake of conciseness, to evaluate the selfenergies at $D=4$ for generic momentum (the procedure is straightforward). We do not use on-shell renormalization: M_W and M_Z are dummy parameters as well as c and s. The massless photon is a source of some infrared problems in the Landau gauge. In Appendix A the analytical tools in order to handle these difficulties are provided (see eqs.(103-106)).

3.1 The $D=4$ amplitudes

The $D=4$ amplitudes are recovered as the finite parts in the Laurent expansion of the generic dimensional regularized amplitudes, normalized by the factor

$$\Lambda^{-(D-4)}. \tag{26}$$

This point has been discussed at length in Refs. [6], [8]. In this procedure we encounter essentially the following cases.

$$\Lambda^{-(D-4)}\Delta_m\Big|_{D\sim 4} = \frac{m^2}{(4\pi)^2}\left(\frac{2}{D-4} - 1 + \gamma + \ln\left[\frac{m^2}{4\pi\Lambda^2}\right]\right), \tag{27}$$

$$\Lambda^{-(D-4)}H(m,M)(p^2)\Big|_{D\sim 4} = \frac{i}{(4\pi)^2}\left\{\frac{2}{D-4} + \gamma \right.$$
$$\left. - \ln(4\pi) + \int_0^1 dx \ln\left(\frac{m^2}{\Lambda^2}(1-x) + \frac{M^2}{\Lambda^2}x - \frac{p^2}{\Lambda^2}x(1-x)\right)\right\}. \tag{28}$$

In Appendix A we give the value of the last integral in eq. (28).

$$\Lambda^{-(D-4)}G(M)(p^2)\Big|_{D\sim 4} = -\frac{i}{(4\pi)^2}\left[\frac{1}{p^2-M^2}\left(\frac{2}{D-4} + \gamma - \ln 4\pi\right)\right.$$
$$\left. + \frac{1}{p^2-M^2}\left(\frac{M^2}{p^2}\ln\frac{[M^2-p^2]}{M^2} + \ln\frac{[M^2-p^2]}{\Lambda^2}\right)\right]. \tag{29}$$

At $D \sim 4$ we use

$$\frac{\Gamma(\frac{D}{2}-2)\Gamma(\frac{D}{2}-2)}{\Gamma(D-4)} \simeq \frac{4}{D-4} + \mathcal{O}((D-4)^2). \tag{30}$$

Then

$$\Lambda^{4-D}\frac{\partial}{\partial M^2}G(M)\Big|_{M=0} \sim -\frac{i}{(4\pi)^2}[-p^2]^{-2}\left[1 + (\gamma-1)(\frac{D}{2}-2)\right]$$
$$\left[1 + (\frac{D}{2}-2)\log\left(-\frac{p^2}{(4\pi)\Lambda^2}\right)\right]\left[\frac{4}{D-4} + \mathcal{O}((D-4)^2)\right]$$
$$= -\frac{2i}{(4\pi)^2}[-p^2]^{-2}\left\{\frac{2}{D-4} - 1 + \gamma + \log\left(-\frac{p^2}{(4\pi)\Lambda^2}\right)\right\}. \tag{31}$$

All other limit expressions can be reduced to eqs. (27-29)

3.2 The counterterms

The counterterms are given by the pole parts of the same Laurent expansion taken with a minus sign and finally multiplied by the common factor $\Lambda^{(D-4)}$. In the expression (29) the pole in $D-4$ dangerously multiplies a nonlocal term. However we shall find that $G(M)$ always enter with a factor $p^2 - M^2$.

4 WW selfenergy

We first list the contributions to the transverse part

4.1 Transverse WW-selfenergy

The Goldstone bosons contribution to the transverse part of Σ_{WW}

$$
\begin{aligned}
& i\Sigma^{\mathrm{GOLDSTONE}}_{TWW} = -i\frac{\Delta_{M_Z}}{4(D-1)}M_W^2\left[\frac{g'^2+\kappa G^2}{G}\right]^2(\frac{1}{M_Z^2}+\frac{1}{p^2}) \\
& +\frac{G(0)}{D-1}M_W^2\left[\frac{gg'}{G}\right]^2\frac{p^2}{4}+\frac{H(0,0)}{4(D-1)G^2M_Z^2}\left\{-p^2M_W^2\left(g'^2+\kappa G^2\right)^2\right. \\
& \left.+g^2M_Z^2\left[2(-3+2D)g'^2M_W^2+G^2p^2(1+\kappa)\right]\right\} \\
& +\frac{H(0,M_Z)}{4G^2}\left[g'^2+\kappa G^2\right]^2M_W^2\left(4+\frac{(M_Z^2-p^2)^2}{(D-1)M_Z^2p^2}\right)
\end{aligned}
\tag{32}
$$

The Faddev-Popov contribution

$$
i\Sigma^{\mathrm{FP}}_{TWW} = -\frac{g^2}{2(D-1)}p^2H(0,0). \tag{33}
$$

The vector boson tadpole contribution

$$
i\Sigma^{\mathrm{TADPOLE}}_{TWW} = i\frac{(D-1)^2}{D}g^2(\Delta_{M_W}+c^2\Delta_{M_Z}). \tag{34}
$$

The γW loop

$$
\begin{aligned}
& i\Sigma^{\gamma W}_{TWW} = -G(0)\frac{g^2g'^2p^6}{4(D-1)G^2M_W^2}-H(0,0)\frac{(D-2)g^2g'^2p^4}{(D-1)G^2M_W^2} \\
& +G(M_W)\frac{g^2g'^2}{4(D-1)G^2M_W^2p^2}\left(M_W^2-p^2\right)^2\left[M_W^4\right. \\
& \left.+2(2D-3)p^2M_W^2+p^4\right] \\
& +H(0,M_W)\frac{g^2g'^2}{(D-1)G^2M_W^2p^2}\left(M_W^2+p^2\right)\left[(D-2)M_W^4\right. \\
& \left.+2(3-2D)p^2M_W^2+(D-2)p^4\right] \\
& -i\Delta_{M_W}\frac{g^2g'^2}{4(D-1)DG^2M_W^2p^2}\Big(D(4D-7)M_W^4+2\Big[D(2D-1) \\
& -2\Big]p^2M_W^2+D(4D-7)p^4\Big)
\end{aligned}
\tag{35}
$$

The ZW loop

$$
\begin{aligned}
&i\Sigma_{TWW}^{ZW} = \frac{g^4 H(0,0)p^6}{4(D-1)G^2 M_W^2 M_Z^2} \\
&-\frac{g^4 H(0,M_W)}{4(D-1)G^2 M_W^2 M_Z^2 p^2}\left(M_W^2-p^2\right)^2\left[M_W^4+2(2D-3)p^2 M_W^2+p^4\right] \\
&-\frac{g^4 H(0,M_Z)}{4(D-1)G^2 M_W^2 M_Z^2 p^2}\left(M_Z^2-p^2\right)^2\left[M_Z^4+2(2D-3)p^2 M_Z^2+p^4\right] \\
&+\frac{i\Delta_{M_Z} g^4}{4(D-1)DG^2 M_Z^2 p^2}\Big\{-D(4D-7)p^4+\Big(D(4D-7)M_W^2 \\
&-2\Big[D(2D-1)-2\Big]M_Z^2\Big)p^2+D\Big[M_W^4+(4D-7)M_Z^2 M_W^2 \\
&+(7-4D)M_Z^4\Big]\Big\} \\
&-\frac{i\Delta_{M_W} g^4}{4(D-1)DG^2 M_W^2 p^2}\Big\{D(4D-7)p^4+\Big(\left(4D^2-2D-4\right)M_W^2 \\
&+(7-4D)DM_Z^2\Big)p^2+D\Big[(4D-7)M_W^4-(4D-7)M_Z^2 M_W^2 \\
&-M_Z^4\Big]\Big\} \\
&+\frac{g^4 H(M_Z,M_W)}{4(D-1)G^2 M_W^2 M_Z^2 p^2}((M_W-M_Z)^2-p^2)((M_W+M_Z)^2-p^2) \\
&\left\{M_W^4+2(2D-3)\left(M_Z^2+p^2\right)M_W^2+M_Z^4+p^4+2(2D-3)M_Z^2 p^2\right\}
\end{aligned} \tag{36}
$$

The charged lepton contribution to transverse Σ_{TWW} is obtained from eq. (23) by using

$$
AS-BT=0, \qquad AS+BT=\frac{g^2}{4}. \tag{37}
$$

Therefore

$$
\begin{aligned}
i\Sigma_{TWW}^{\mathrm{LEPTONS}} &= \frac{g^2}{2}\frac{1}{D-1}\sum_{l=e,\mu,\tau}\left\{(2-D)i\Delta_{M_l}+\frac{{M_l}^2}{p^2}i\Delta_{M_l}\right. \\
&\left.+H(0,M_l)(-p^2+{M_l}^2)\left[(2-D)-\frac{{M_l}^2}{p^2}\right]\right\}.
\end{aligned} \tag{38}
$$

The quarks contribution to the transverse part of Σ_{WW} is given by eq. (23) where

$$
AS-BT=0, \qquad (AS+BT)_{ab}=\frac{g^2}{4}3V_{ab}V_{ab}^* \tag{39}
$$

being V_{ab} the CKM matrix. Thus

$$
\begin{aligned}
i\Sigma_{TWW}^{\text{QUARKS}} &= 3\frac{g^2}{2(D-1)}\sum_{ab} V_{ab}V_{ab}^* \Bigg\{ i\Delta_{m_a}\left((2-D)+\frac{1}{p^2}({m_a}^2-{M_b}^2)\right) \\
&+ i\Delta_{M_b}\left((2-D)-\frac{1}{p^2}({m_a}^2-{M_b}^2)\right) \\
&+ H(m_a,M_b)\Bigg[(2-D)(-p^2+{M_b}^2+{m_a}^2) \\
&- \frac{1}{p^2}\left(({m_a}^2-{M_b}^2)^2-p^2({m_a}^2+{M_b}^2)\right)\Bigg]\Bigg\}
\end{aligned}
\tag{40}
$$

4.2 Longitudinal WW selfenergy

In a similar way we list the contributions for the longitudinal parts of Σ_{LWW}. The Goldstone bosons contribution to the longitudinal part of Σ_{WW}

$$
\begin{aligned}
i\Sigma_{LWW}^{\text{GOLDSTONE}} &= i\Delta_{M_Z}M_W^2\frac{\left(\kappa G^2+g'^2\right)^2}{4G^2}\left(\frac{1}{p^2}+\frac{1}{M_Z^2}\right) \\
&- G(0)M_W^2\frac{g^2g'^2p^2}{4G^2} - H(0,M_Z)\frac{\left(\kappa G^2+g'^2\right)^2 M_W^2}{4G^2M_Z^2p^2}\left(M_Z^2-p^2\right)^2 \\
&+ \frac{1}{4}H(0,0)\left\{\frac{M_W^2}{G^2M_Z^2}\left[2g^2g'^2M_Z^2+\left(\kappa G^2+g'^2\right)^2p^2\right]-\frac{g^2\kappa^2p^2}{\kappa+1}\right\}.
\end{aligned}
\tag{41}
$$

The Faddeev-Popov bosons contribution to the longitudinal part of Σ_{WW}

$$
i\Sigma_{LWW}^{\text{FP}} = -\frac{g^2}{2}p^2H(0,0). \tag{42}
$$

The vector boson tadpole contribution to longitudinal part

$$
i\Sigma_{LWW}^{\text{TADPOLE}} = i\frac{(D-1)^2}{D}g^2(\Delta_{M_W}+c^2\Delta_{M_Z}). \tag{43}
$$

The γW loop contribution to longitudinal part

$$
\begin{aligned}
i\Sigma_{LWW}^{\gamma W} &= -G(W)\frac{g^2g'^2M_W^2}{4G^2p^2}\left(M_W^2-p^2\right)^2 \\
&- i\Delta_{M_W}\frac{g^2g'^2}{2G^2p^2}\left[\left(\frac{7}{2}-2D\right)M_W^2+\left(2D-3+\frac{2}{D}\right)p^2\right] \\
&+ H(0,M_W)\frac{g^2g'^2}{2G^2p^2}\left[-2(D-2)M_W^4-p^2M_W^2+p^4\right]
\end{aligned}
\tag{44}
$$

The ZW loop contribution to longitudinal part

$$
i\Sigma_{LWW}^{ZW} =
$$

$$\frac{H(0,M_W)g^4}{4G^2M_Z^2p^2}\left(M_W^3-M_Wp^2\right)^2+\frac{H(0,M_Z)g^4}{4G^2M_W^2p^2}\left(M_Z^3-M_Zp^2\right)^2$$
$$-\frac{i\Delta_{M_Z}g^4}{2G^2p^2}\Bigg\{\frac{M_W^2}{2M_Z^2}(M_W^2-p^2)+\left(2D-3+\frac{2}{D}\right)p^2$$
$$+\left(2D-\frac{7}{2}\right)(M_W^2-M_Z^2)\Bigg\}$$
$$-\frac{i\Delta_{M_W}g^4}{2G^2p^2}\Bigg\{\frac{M_Z^2}{2M_W^2}(M_Z^2-p^2)+\left(2D-3+\frac{2}{D}\right)p^2$$
$$+\left(2D-\frac{7}{2}\right)\left(M_Z^2-M_W^2\right)\Bigg\}$$
$$-\frac{H(M_Z,M_W)g^4}{4G^2M_W^2M_Z^2p^2}\left(M_W^2-M_Z^2\right)^2\Bigg\{M_W^4+\left((4D-6)M_Z^2-2p^2\right)M_W^2$$
$$+\left(M_Z^2-p^2\right)^2\Bigg\} \tag{45}$$

The charged lepton contribution to longitudinal Σ_{LWW} is obtained from eqs. (37) and (24)

$$i\Sigma_{LWW}^{\text{LEPTONS}}=\frac{g^2}{2p^2}\sum_{l=e,\mu,\tau}\left\{-iM_l^2\Delta_{M_l}+H(0,M_l)(-p^2+{M_l}^2){M_l}^2\right\} \tag{46}$$

The quark contribution to longitudinal Σ_{LWW} is obtained from eqs. (39) and (24)

$$i\Sigma_{LWW}^{\text{QUARKS}}=-3\frac{g^2}{2p^2}\sum_{ab}V_{ab}V_{ab}^*\Bigg\{i\Delta_{m_a}({m_a}^2-{M_b}^2)$$
$$-i\Delta_{M_b}({m_a}^2-{M_b}^2)$$
$$-H(m_a,M_b)\left[({m_a}^2-{M_b}^2)^2-p^2({m_a}^2+{M_b}^2)\right]\Bigg\} \tag{47}$$

5 ZZ selfenergy

We first list the contributions to the transverse part.

5.1 The transverse ZZ selfenergy

The Goldstone contribution to the transverse part of ZZ selfenergy

$$i\Sigma_{TZZ}^{\text{GOLDSTONE}}$$

$$= -i\Delta_{M_W} \frac{\left(M_W^2+p^2\right)\left(\kappa G^2+g'^2\right)^2}{2(D-1)G^2p^2}$$
$$+H(0,M_W)\frac{\left(\kappa G^2+g'^2\right)^2}{2(D-1)G^2p^2}\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right]$$
$$+H(0,0)\frac{p^2}{4(D-1)G^2}\left[g^4-2\left(\kappa G^2+g'^2\right)g^2-\left(\kappa G^2+g'^2\right)^2\right] \quad (48)$$

The Faddev-Popov contribution to the transverse part of ZZ selfenergy

$$i\Sigma_{TZZ}^{\mathrm{FP}} = -\frac{g^4}{G^2}\frac{1}{2(D-1)}p^2H(0,0). \quad (49)$$

The vector boson tadpole contribution to transverse part

$$i\Sigma_{TZZ}^{\mathrm{TADPOLE}} = i\frac{g^4}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W}. \quad (50)$$

The WW loop contribution to transverse part of ZZ selfenergy

$$i\Sigma_{TZZ}^{WW} =$$
$$H(0,0)\frac{g^4p^6}{4(D-1)G^2M_W^4}$$
$$-H(M_W,M_W)\frac{g^4\left(4M_W^2-p^2\right)}{4(D-1)G^2M_W^4}\left[4(D-1)M_W^4+4(2D-3)p^2M_W^2+p^4\right]$$
$$-H(0,M_W)\frac{g^4\left(M_W^2-p^2\right)^2}{2(D-1)G^2M_W^4p^2}\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right]$$
$$-i\Delta_{M_W}\frac{g^4}{2(D-1)DG^2M_W^2p^2}\left[-DM_W^4+(5D-4)p^2M_W^2+D(4D-7)p^4\right] \quad (51)$$

The neutrino contributions to transverse Σ_{TZZ} is obtained from eq. (23) by using

$$AS-BT=0, \qquad AS+BT=\frac{G^2}{8}. \quad (52)$$

Therefore the neutrinos yield

$$i\Sigma_{TZZ}^{\nu\nu} = -3p^2\frac{G^2}{4}\frac{(2-D)}{D-1}H(0,0) \quad (53)$$

For the charged leptons as well for the up- and down-quarks the contribution to the selfenergy Σ_{TZZ} has the same form (23)

$$i\Sigma_{TZZ}^{CHARGED\ FERMIONS} = \sum_j 4\frac{1}{D-1}\Bigg\{i(AS+BT)(2-D)\Delta_{m_j}$$
$$+H(m_j,m_j)\left[2{m_j}^2\Bigg((2-D)BT+AS\Bigg)-p^2\frac{(2-D)}{2}(AS+BT)\right]\Bigg\} \quad (54)$$

where the sum is over the flavors. For the leptons

$$AS - BT = G^2 s^2[-\frac{1}{2} + s^2]$$
$$AS + BT = G^2[\frac{1}{8} - \frac{1}{2}s^2 + s^4]. \tag{55}$$

For up quarks

$$AS - BT = 3G^2 s^2[-\frac{1}{2}Q_u + s^2 Q_u^2]$$
$$AS + BT = 3G^2[\frac{1}{8} - \frac{1}{2}s^2 Q_u + s^4 Q_u^2], \qquad Q_u = \frac{2}{3}. \tag{56}$$

For the down quarks

$$AS - BT = 3G^2 s^2[\frac{1}{2}Q_d + s^2 Q_d^2]$$
$$AS + BT = 3G^2[\frac{1}{8} + \frac{1}{2}s^2 Q_d + s^4 Q_d^2], \qquad Q_d = -\frac{1}{3}. \tag{57}$$

5.2 The longitudinal ZZ selfenergy

Now the longitudinal part of ZZ selfenergy.

The Goldstone contribution to the longitudinal part of ZZ selfenergy

$$i\Sigma_{LZZ}^{\text{GOLDSTONE}} = i\Delta_{M_W}\frac{\left(\kappa G^2 + g'^2\right)^2}{2G^2 p^2}\left(M_W^2 + p^2\right)$$
$$+H(0,0)\frac{\left(\kappa G^2 + g'^2\right)^2}{2G^2}p^2 - H(0,M_W)\frac{\left(\kappa G^2 + g'^2\right)^2}{2G^2 p^2}\left(M_W^2 - p^2\right)^2 \tag{58}$$

The Faddev-Popov contribution to the longitudinal part of ZZ selfenergy

$$i\Sigma_{LZZ}^{\text{FP}} = -\frac{g^2 c^2}{2}p^2 H(0,0). \tag{59}$$

The vector boson tadpole contribution to the longitudinal part of ZZ selfenergy

$$i\Sigma_{LZZ}^{\text{TADPOLE}} = i\frac{g^4}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W}. \tag{60}$$

The WW loop contribution to the longitudinal part of ZZ selfenergy

$$i\Sigma_{LZZ}^{WW} = H(0,M_W)\frac{g^4}{2G^2 p^2}\left(M_W^2 - p^2\right)^2$$
$$-i\Delta_{M_W}\frac{g^4}{2G^2 p^2}\left\{M_W^2 + \frac{1}{D}\left[D(4D-7)+4\right]p^2\right\}. \tag{61}$$

The fermion contribution to the longitudinal parts of Σ_{ZZ} is given by

$$i\Sigma_{LZZ}^{\text{FERMIONS}} = -8BT \sum_{j=leptons,quarks} m_j^2 H(m_j,m_j). \tag{62}$$

where B, T is taken from eqs. (55), (56) and (57).

6 $\gamma\gamma$ selfenergy

We first list the contributions to the transverse part.

6.1 The transverse $\gamma\gamma$ selfenergy

The Goldstone contribution to the transverse part of $\gamma\gamma$ selfenergy

$$
\begin{aligned}
i\Sigma^{\rm GOLD}_{T\gamma\gamma} &= -i\Delta_{M_W}\frac{g^2g'^2}{2(D-1)G^2p^2}\left(M_W^2+p^2\right)+H(0,0)\frac{g^2p^2g'^2}{2(D-1)G^2} \\
&+H(0,M_W)\frac{g^2g'^2}{2(D-1)G^2p^2}\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right] \qquad (63)
\end{aligned}
$$

The Faddev-Popov contribution to the transverse part of $\gamma\gamma$ selfenergy

$$
i\Sigma^{\rm FP}_{T\gamma\gamma} = -\frac{e^2}{2(D-1)}p^2H(0,0). \qquad (64)
$$

The vector boson tadpole contribution to transverse part of $\gamma\gamma$ selfenergy

$$
i\Sigma^{\rm TADPOLE}_{T\gamma\gamma} = i\frac{g^2{g'}^2}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W} \qquad (65)
$$

The WW loop contribution to transverse part of $\gamma\gamma$ selfenergy

$$
\begin{aligned}
i\Sigma^{WW}_{T\gamma\gamma} &= H(0,0)\frac{g^2g'^2p^6}{4(D-1)G^2M_W^4}-H(M_W,M_W)\frac{g^2g'^2}{4(D-1)G^2M_W^4} \\
&\left(4M_W^2-p^2\right)\left[4(D-1)M_W^4+4(2D-3)p^2M_W^2+p^4\right] \\
&-H(0,M_W)\frac{g^2g'^2}{2(D-1)G^2M_W^4p^2}\left(M_W^2-p^2\right)^2\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right] \\
&-i\Delta_{M_W}\frac{g^2g'^2}{2(D-1)DG^2M_W^2p^2}\left[-DM_W^4+(5D-4)p^2M_W^2+D(4D-7)p^4\right] \\
&\qquad (66)
\end{aligned}
$$

The electromagnetic interaction gives

$$
AS-BT=eQ, \qquad AS+BT=eQ \qquad (67)
$$

then

$$
\begin{aligned}
i\Sigma^{\rm FERMION}_{T\gamma\gamma} &= 4\frac{e^2}{D-1}\sum_{j=l,q,{\rm color}}Q^2\Bigg\{i(2-D)\Delta_{m_j} \\
&+\frac{-p^2(2-D)+4m_j^2}{2}H(m_j,m_j)\Bigg\}. \qquad (68)
\end{aligned}
$$

For small p^2 one gets

$$
i\Sigma^{\rm FERMION}_{T\gamma\gamma} = \mathcal{O}(p^2). \qquad (69)
$$

6.2 The longitudinal $\gamma\gamma$ selfenergy.

The Goldstone contribution to the longitudinal part of $\gamma\gamma$ selfenergy

$$i\Sigma_{L\gamma\gamma}^{\text{GOLDSTONE}} = -\frac{1}{2p^2}M_W^2\left[\frac{gg'}{G}\right]^2$$
$$\left[M_W^2(\frac{p^2}{M_W^2}-1)^2H(0,M_W) - \frac{p^4}{M_W^2}H(0,0) - i\Delta_{M_W}(\frac{p^2}{M_W^2}+1)\right] \quad (70)$$

The Faddev-Popov contribution to the longitudinal part of $\gamma\gamma$ selfenergy

$$i\Sigma_{L\gamma\gamma}^{\text{FP}} = -\frac{e^2}{2}p^2H(0,0). \quad (71)$$

The vector boson tadpole contribution to the longitudinal part of $\gamma\gamma$ selfenergy

$$i\Sigma_{L\gamma\gamma}^{\text{TADPOLE}} = i\frac{g^2g'^2}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W} \quad (72)$$

The WW loop contribution to the longitudinal part of $\gamma\gamma$ selfenergy

$$i\Sigma_{L\gamma\gamma}^{WW} = \frac{g^2g'^2}{G^2p^2}\left\{-i\Delta_{M_W}\left[-\frac{p^2}{2}+\frac{{M_W}^2}{2}+\frac{2p^2}{D}+2Dp^2-3p^2\right]\right.$$
$$\left.+\frac{1}{2}H(0,M_W)(p^2-{M_W}^2)^2\right\}. \quad (73)$$

For the longitudinal part we get

$$i\Sigma_{L\gamma\gamma}^{\text{FERMION}} = 0. \quad (74)$$

It is remarkable that the sum of all the contributions (70)-(73) amounts to zero photon longitudinal self-energy. This is in agreement with the Ward identity for QED derived in Ref. [1]

7 $Z\gamma$ selfenergy

We first list the contributions to the transverse part

7.1 The transverse $Z\gamma$ selfenergy

The Goldstone contribution to the transverse part of $Z\gamma$ selfenergy

$$i\Sigma_{TZ\gamma}^{\text{GOLDSTONE}}$$

$$= H(0,0)\frac{g'g^3p^2}{2(D-1)G^2} + i\Delta_{M_W}\frac{gg'\left(kG^2+g'^2\right)}{2(D-1)G^2p^2}\left(M_W^2+p^2\right)$$
$$-H(0,M_W)\frac{gg'\left(kG^2+g'^2\right)}{2(D-1)G^2p^2}\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right] \quad (75)$$

The Faddev-Popov contribution to the transverse part of $Z\gamma$ selfenergy

$$i\Sigma_{TZ\gamma}^{\mathrm{FP}} = -\frac{g^3g^{'}}{G^2}\frac{1}{2(D-1)}p^2H(0,0) \quad (76)$$

The vector boson tadpole contribution to transverse part of $Z\gamma$ selfenergy

$$i\Sigma_{TZ\gamma}^{\mathrm{TADPOLE}} = i\frac{g^3g^{'}}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W} \quad (77)$$

The WW loop contribution to transverse part of $Z\gamma$ selfenergy

$$\begin{aligned}
i\Sigma_{TZ\gamma}^{WW} &= H(0,0)\frac{g^3g'p^6}{4(D-1)G^2M_W^4} \\
&-H(M_W,M_W)\frac{g^3g'}{4(D-1)G^2M_W^4}\left(4M_W^2-p^2\right) \\
&\left[4(D-1)M_W^4+4(2D-3)p^2M_W^2+p^4\right] \\
&-H(0,M_W)\frac{g^3g'}{2(D-1)G^2M_W^4p^2}\left(M_W^2-p^2\right)^2\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right] \\
&-i\Delta_{M_W}\frac{g^3g'}{2(D-1)DG^2M_W^2p^2}\left[-DM_W^4+(5D-4)p^2M_W^2+D(4D-7)p^4\right]
\end{aligned} \quad (78)$$

The fermion contribution to the transverse part of $\Sigma_{Z\gamma}$ is

$$i\Sigma_{TZ\gamma}^{\mathrm{FERMION}} = \sum_{j=leptons,quarks,color} 4\frac{(AS)_j}{D-1}\left\{i(2-D)\Delta_{m_j}\right.$$
$$\left.+H(m_j,m_j)\left(p^2\frac{(D-2)}{2}+2{m_j}^2\right)\right\} \quad (79)$$

where the constants A,B,S,T are: Neutrinos

$$AS-BT=0, \qquad AS+BT=0 \quad (80)$$

Charged leptons

$$AS-BT=AS+BT=eG[\frac{1}{4}-s^2] \quad (81)$$

Up-Quarks

$$AS - BT = AS + BT = eQ_uG[\frac{1}{4} - s^2Q_u] \qquad Q_u = \frac{2}{3} \tag{82}$$

Down-Quarks

$$AS - BT = AS + BT = -eQ_dG[\frac{1}{4} + s^2Q_d] \qquad Q_d = -\frac{1}{3} \tag{83}$$

7.2 The longitudinal $Z\gamma$ selfenergy

The Goldstone contribution to the longitudinal part of $Z\gamma$ selfenergy

$$i\Sigma^{\text{GOLDSTONE}}_{LZ\gamma} = \frac{1}{2}\frac{1}{p^2}\left[M_W^2\left(\frac{g^{'2}}{G} + \kappa G\right)\frac{gg^{'}}{G}\right]$$
$$\left\{M_W^2(\frac{p^2}{M_W^2} - 1)^2 H(0, M_W) - \frac{p^4}{M_W^2}H(0,0) - i\Delta_{M_W}(\frac{p^2}{M_W^2} + 1)\right\} \tag{84}$$

The Faddev-Popov contribution to the longitudinal part of $Z\gamma$ selfenergy

$$i\Sigma^{\text{FP}}_{TZ\gamma} = -\frac{g^3g^{'}}{G^2}\frac{1}{2}p^2H(0,0) \tag{85}$$

The vector boson tadpole contribution to the longitudinal part of $Z\gamma$ selfenergy

$$i\Sigma^{\text{TADPOLE}}_{LZ\gamma} = i\frac{g^3g^{'}}{G^2}2\frac{(D-1)^2}{D}\Delta_{M_W} \tag{86}$$

The WW loop contribution to the longitudinal part of γZ selfenergy

$$i\Sigma^{WW}_{LZ\gamma} = \frac{g^3g^{'}}{G^2p^2}\left\{-i\Delta_{M_W}\left[-\frac{p^2}{2} + \frac{{M_W}^2}{2} + \frac{2p^2}{D} + 2Dp^2 - 3p^2\right]\right.$$
$$\left. + \frac{1}{2}H(0, M_W)(p^2 - {M_W}^2)^2\right\} \tag{87}$$

For the longitudinal one has

$$i\Sigma^{\text{FERMION}}_{LZ\gamma} = 0 \tag{88}$$

8 Physical unitarity for diagonal elements

It is simple to trace, at the one-loop level, the contributions due to unphysical modes (Faddeev-Popov ghosts, Goldstone bosons and scalar parts of the vector mesons). These contributions have to cancel when we evaluate the transverse part of the selfenergies on-shell. Here we show that this cancellation works in generic D dimension when the transverse part is taken on-shell.

8.1 Σ_{TWW}: the unphysical $H(0,0), H(0,M_Z)$ and $G(M_W), G(0)$

We collect the terms in the self-energy in eqs. (32)-(36) proportional to $H(0,0)$ and $H(0,M_Z)$, in order to check physical unitarity. They must vanish at $p^2 = M_W^2$ since they are due to the presence of unphysical modes (Goldstone and longitudinal part of the vector bosons). We get

$$\begin{aligned} & H(0,0)\frac{g^2(p^2-M_W^2)}{4(D-1)G^2(k+1)M_W^4}\Big\{g^4p^2(p^2+M_W^2) \\ & -2g^2g'^2(1+k)M_W^2\Big[(-3+2D)M_W^2+2(-2+D)p^2\Big] \\ & -2g'^4(1+k)M_W^2\Big[(-3+2D)M_W^2+2(-2+D)p^2\Big]\Big\} \end{aligned} \tag{89}$$

and

$$\begin{aligned} & -H(0,M_Z)\frac{(M_W^2-p^2)}{4(D-1)G^4(k+1)M_W^4p^2} \\ & \Big[G^4(k+1)^2M_W^4+2(2D-3)g^2G^2(k+1)p^2M_W^2+g^4p^4\Big] \\ & \Big\{\left[(2k+1)M_W^2-p^2\right]g^2+2g'^2(k+1)M_W^2\Big\}. \end{aligned} \tag{90}$$

Thus they are zero on-shell. Similarly one can prove that on-shell $p^2 = M_W^2$ the coefficients of $G(0)$ and of $G(M_W)$ are zero in generic D dimensions.

8.2 Σ_{TZZ} terms proportional to $H(0,0)$ and $H(0,M_W)$

We collect the terms in the self-energy in eqs. (48)-(51) proportional to $H(0,0)$ and $H(0,M_W)$.

$$\frac{H(0,0)}{D-1}\frac{p^2}{4M_W^4}\frac{g^4}{G^2}\Big\{p^4-M_Z^4\Big\}. \tag{91}$$

and similarly

$$\begin{aligned} & \frac{g^4H(0,M_W)}{2(D-1)G^2M_W^4p^2}\left(p^2-M_Z^2\right) \\ & \left(2M_W^2-M_Z^2-p^2\right)\left[M_W^4+2(2D-3)p^2M_W^2+p^4\right]. \end{aligned} \tag{92}$$

Thus again physical unitarity is working for the self-mass of Z.

8.3 $\Sigma_{T\gamma\gamma}$: the Limit $p^2 = 0$

We collect the terms in the self-energy in eqs. (63)-(66) proportional to $H(0,0)$ and then we put at $p^2 = 0$ in order to check that the photon remains with zero mass. One verifies that

$$\frac{H(0,0)p^2}{D-1}\frac{g^2 g'^2}{G^2}\left\{1-\frac{1}{2}+\frac{p^4}{4M_W^4}-\frac{1}{2}\right\}\Bigg|_{p^2=0} \quad = \quad 0. \tag{93}$$

For the terms involving $H(0,M)$ one needs the identity

$$H(0,M) = \frac{i\Delta_M}{M^2}\left[1-\frac{p^2}{M^2}(D-4)\right]+\mathcal{O}(p^4). \tag{94}$$

It is then straightforward to verify that

$$\lim_{p^2=0} i\Sigma_{T\gamma\gamma} = 0. \tag{95}$$

Thus the mass of the photon remains null.

8.4 Unitarity for the $\Sigma_{TZ\gamma}$

The unitarity properties of $\Sigma_{TZ\gamma}$ are strictly connected to the process where this graph contributes (e.g. $Z \to l + \bar{l}$). Thus more graphs are necessary in order to verify physical unitarity. This subject is outside the scope of the present work.

9 W and Z selfmasses

By using the procedure of extracting the finite parts from the D-dimensional amplitudes described in Sec. 3 we evaluate the selfmasses for W and Z bosons. Since we have already thoroughly examined the properties of the amplitudes in D dimensions at the onshell momenta, the selfmasses can be evaluated by any computer algorithm. We do not reproduce the result in the present paper.

10 Parameters fit

In this section we provide an estimate of the parameters introduced in the model. The parameters g, g', M can be fixed by experiments that are essentially at low momentum transfer as for instance: α, G_μ and the $\nu - e$

scattering that provides a precise value of $\sin\theta_W$. Our calculation of the selfenergies can be checked on the physics of the vector bosons W, Z. The physical masses are the imput for the determination of the extra parameters of the model: κ and Λ.

For the processes at nearly zero momentum transfer we can use Particle Data Group [16] values as

$$\begin{aligned} \alpha &= 1/137.0599911(46) \\ G_\mu &= 1.16637(1)\cdot 10^{-5} GeV \end{aligned} \tag{96}$$

and from $\nu - e$ scattering [17]

$$\sin^2\theta_W = 0.2324 \pm 0.011. \tag{97}$$

We get:

$$\begin{aligned} g &= \frac{e}{s} = \frac{\sqrt{4\pi\alpha}}{s} = 0.6281 \\ g' &= \frac{e}{c} = 0.3456 \\ M &= \sqrt{\frac{1}{4\sqrt{2}G_F}} = 123.11\ GeV. \end{aligned} \tag{98}$$

With these inputs we can evaluate the values for the other two parameters by imposing the conditions on the mass corrections

$$\begin{aligned} (gM)^2 + \Delta M_W^2 &= (80.428 \pm 0.039)^2\ GeV^2 \\ M^2G^2(1+\kappa) + \Delta M_Z^2 &= (91.1876 \pm 0.0021)^2\ GeV^2. \end{aligned} \tag{99}$$

One gets

$$\begin{aligned} \kappa &= 0.0085 \\ \Lambda &= 283\ GeV. \end{aligned} \tag{100}$$

The widths of the vector mesons obtained from the imaginary parts of the self-energies (all fermions are taken massless but the top with $M_{top} = 174.2\ GeV$) are

$$\begin{aligned} \Gamma_Z &= 2.203\ GeV \quad (\text{exp.} \quad (2.4952 \pm 0.0023)\ GeV) \\ \Gamma_W &= 1.818\ GeV \quad (\text{exp.} \quad (2.141 \pm 0.041)\ GeV)\,. \end{aligned} \tag{101}$$

These values are quite encouraging for the calculation of further radiative corrections. However one should consider only the order of magnitude of these numbers. In fact they depend strongly from the value of $\sin^2\theta_W$. Only a fit including other sensitive quantities will be able to reduce their variability.

11 Conclusions

The one loop evaluation of selfenergies for the vector mesons in the Electroweak model based on nonlinearly realized gauge group has been explicitly performed in D dimensions. The finite amplitudes in $D = 4$ has been achieved according to the procedure suggested by the local functional equation associated to the local invariance of the path integral measure. In practice this implies the minimal subtraction of poles in $D - 4$ on properly normalized amplitudes. Thus in the model the Higgs sector is absent and the parameters are fixed by the classical lagrangian (no free parameters for the counterterms and therefore no on-shell renormalization). Two new parameters appear: a second mass term parameter and a scale of radiative corrections. The Spontaneous Symmetry Breaking parameter v is not a physical constant. The scheme is very rigid and it should be checked by the comparison with the experimental measures.

The calculation has been performed in the Landau gauge and by using the symmetric formalism whenever it was possible. We checked the physical unitarity and the absence of v in the measurable quantities.

A very simple evaluation has been performed for the parameters of the classical action, by using leptonic processes. The parameter that describes the departure from the Weinberg relation between M_W and M_Z is very small and the scale of the radiative corrections is of the order of hundred GeV. That means that the model is on solid grounds and it is reasonable to make further efforts for the evaluation of the radiative corrections in other processes.

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A Limit $D = 4$ for the logarithmic integral

We collect in this Appendix some relevant formulas.

$$\begin{aligned} \Delta_m &\equiv \frac{1}{(2\pi)^D} \int d^D q \frac{i}{q^2 - m^2} \,, \\ H(m,M) &\equiv -\frac{1}{(2\pi)^D} \int d^D q \frac{1}{q^2 - m^2} \frac{1}{(p+q)^2 - M^2} \,. \end{aligned} \tag{102}$$

The following identities allow to prove the cancellation of infrared divergences due to the massless photon.

$$\begin{aligned} G(M) &\equiv \frac{\partial}{\partial m^2} H(m,M)\Big|_{m^2=0} \\ &= \frac{i}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(3-\frac{D}{2})}{\Gamma(2)} \int_0^1 dx (1-x)[M^2 x - p^2 x(1-x)]^{\frac{D}{2}-3} \,. \end{aligned} \tag{103}$$

$$\begin{aligned} G(0) &\equiv \lim_{M=0} G(M) \\ &= \frac{i}{(4\pi)^{\frac{D}{2}}} [-p^2]^{\frac{D}{2}-3} \frac{\Gamma(3-\frac{D}{2})}{\Gamma(2)} \frac{\Gamma(\frac{D}{2}-2)\Gamma(\frac{D}{2}-1)}{\Gamma(D-3)} \,. \end{aligned} \tag{104}$$

$$\lim_{m=0} \frac{1}{m^2} \Delta_m = \lim_{m=0} \frac{1}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(1-\frac{D}{2})}{\Gamma(1)} (m^2)^{\frac{D}{2}-2} = 0 \,. \tag{105}$$

$$\begin{aligned} &\frac{\partial}{\partial M^2} G(M)\Big|_{M=0} \\ &= -\frac{i}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(4-\frac{D}{2})}{\Gamma(2)} [-p^2]^{\frac{D}{2}-4} \frac{\Gamma(\frac{D}{2}-2)\Gamma(\frac{D}{2}-2)}{\Gamma(D-4)} \,. \end{aligned} \tag{106}$$

For the integral in eq. (28) use the notation

$$a = p^2, \quad b = -p^2 + M^2 - m^2, \quad c = m^2 \tag{107}$$

i.e.

$$\int_0^1 dx \ln\Big(p^2 x^2 + [M^2 - m^2 - p^2]x + m^2\Big) = \int_0^1 dx \ln\Big(ax^2 + bx + c\Big) . \tag{108}$$

If one or two masses are zero one gets

$$\int_0^1 dx \ln\Big(x[p^2x+M^2-p^2]\Big)$$
$$= -2+\ln\Big|p^2-M^2\Big| + \frac{M^2}{p^2}\ln\Big|\frac{M^2}{p^2-M^2}\Big| - i\pi\frac{p^2-M^2}{p^2}\theta(p^2-M^2) \tag{109}$$

Let

$$\Delta = [m^2+M^2-p^2]^2 - 4m^2M^2. \tag{110}$$

By following the Feynman prescription one gets:
for $0 < p^2 < (M-m)^2$, $\Delta > 0$ and then the integral is

$$-2+\ln(a+b+c)+\frac{b}{2a}\ln\frac{(a+b+c)}{c}+\frac{\sqrt{\Delta}}{2a}\ln\frac{2c+b-\sqrt{\Delta}}{2c+b+\sqrt{\Delta}}; \tag{111}$$

for $p^2 > (M+m)^2$, $\Delta > 0$ and then the integral is

$$-2+\ln(a+b+c)+\frac{b}{2a}\ln\frac{(a+b+c)}{c}+\frac{\sqrt{\Delta}}{2a}\ln\frac{2c+b-\sqrt{\Delta}}{2c+b+\sqrt{\Delta}}-i\frac{\sqrt{\Delta}}{a}; \tag{112}$$

for $p^2 = (M-m)^2$ or $p^2 = (M+m)^2$, $\Delta = 0$ and then the integral is

$$-2+\ln(a+b+c)+\frac{b}{2a}\ln\frac{(a+b+c)}{c}; \tag{113}$$

for $(M-m)^2 < p^2 < (M+m)^2$, $\Delta < 0$ and then the integral is

$$-2-\frac{b}{2a}\ln c+(1+\frac{b}{2a})\ln[a+b+c]$$
$$+\frac{\sqrt{-\Delta}}{a}\Big\{\tan^{-1}\Big(\frac{2a+b}{\sqrt{-\Delta}}\Big)-\tan^{-1}\Big(\frac{b}{\sqrt{-\Delta}}\Big)\Big\} \tag{114}$$

where $-\frac{\pi}{2} < \tan^{-1}(x) < \frac{\pi}{2}, x \in \mathcal{R}$.

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